Push recovery by center of pressure manipulation

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SUMMARY

Standing balance is an important feature of bipedal robots that are usually exposed to external perturbations. The usual objective for a bipedal robot is tracking of desired joint positions which is not well suited for overcoming large disturbances. Instead, it has been shown that some dynamic strategies allow to handle large disturbances. However, despite the advances of non-linear control, most of the methods do not exploit the full dynamics of the system. In this study, we aimed to provide a connection between non-linear controls and balance principles for push recovery. We proposed to use control Lyapunov functions to achieve joint tracking and center of pressure manipulation. The result showed that our simplified model could effectively estimate COP position and handle disturbances in more robust manners.

INTRODUCTION

Standing balance plays an important role for bipedal robots in real environments where unexpected external perturbations can be imposed on the robots at any time. The capacity to resist against the perturbations in bipedal robots is limited by the torque limitations and the center of pressure (COP) location. Whenever the COP lies inside the foot support, robots may have chances to resist against the perturbations and to maintain the balance without a need to make steps to prevent falls. Furthermore, those constraints are highly dependent on the control algorithms. Stephens (Stephens 2007) suggested that direct joint tracking is not beneficial to keep balance against large disturbances. Instead, he proposed a series of strategies aimed to keeping the COP within the foot support by allowing some joints to deviate from their desired trajectories. Hofmann (Hofmann 2006) stated that the manipulation of the horizontal center of mass (COM) is the key factor for stable standing balance and proposed a set of strategies aimed to manipulating COM. However, defining manually the series of actions for stability in response to perturbation is a tedious task and obscures the dynamic relation among all the links that could be used in beneficial ways to avoid falling.

By using nonlinear control techniques, it is possible to dynamically generate balancing strategies with no need to directly and manually design them. Furthermore, a quadratic programming (QP)-based controller allows to combine different control objectives (Morris, Powell, and Ames 2013). These objectives can contribute for both balance and joint tracking during perturbations. Specifically, the objective for joint tracking was achieved using an appropriate control lyapunov function (CLF); the objective for COP regulation was accomplished by use of a simplified model of a biped robot and the definition of a corresponding CLF.

METHODS

Confining the movement in the sagittal plane, a monoped model with foot, shank, thigh and torso was used. The joint coordinates could be expressed as $\Theta = (\theta_{ankle}, \theta_{knee}, \theta_{hip})^T$ by considering the angles of the ankle, knee and hip respectively. Additionally, to fully describe the model, the coordinates that define the pose of the foot are expressed as $\Theta_e = (x, y, \theta_0)^T$. Then, the extended coordinates that define the system can be expressed as $q_e = (\Theta_e^T, \Theta^T)^T$. The dynamics of the system are expressed as follows:

$$D_e(q_e)\ddot{q_e} + C_e(q_e, \dot{q_e})\dot{q_e} + G(q_e) = Bu + J_{sf}^T F = \bar{B}\bar{u} \quad (1)$$

where, $D_e(q_e) \in \mathbb{R}^6$ is the inertial matrix, $C_e(q_e, \dot{q_e}) \in \mathbb{R}^6$ is the Coriolis matrix, $G(q_e) \in \mathbb{R}^{6x1}$ is the gravity vector, $B = I_6$ is the torque map, J_{sf} is the Jacobian of the constraints at the support foot (Sinnet et al. 2011) that keeps it from moving, $F = (F_x, F_y, M_z)$ is the reaction force (F_x, F_y) and moment M_z applied by the floor at the foot base, $\bar{B} = \begin{bmatrix} B & J_{sf}^T \end{bmatrix}$ is the extended map from the torques and reaction forces and $\bar{u} = (u^T, F^T)^T$ the extended input of the system.

The dynamics can be expressed in a affine control system form by choosing the state $x = (q_e^T, \dot{q_e}^T)^T$. Then, the control system is written as $\dot{x} = f(x) + g(x)\bar{u}$

The joint tracking objectives are selected to keep the actuated robot joints at zero, i.e standing position. Therefore, the vector output is defined by the relative degree two output $y_1(q_e) = \begin{bmatrix} \theta_{ankle} & \theta_{knee} & \theta_{hip} \end{bmatrix}^T$, then a feedback linearizing controller can be formulated as below.

$$\ddot{y_1}(q_e) = L_{f^2} y_1(q_e) + L_g L_f y_1(q_e)\bar{u} = \mu \quad (2)$$

To control the zero dynamics conformed by $\eta = (y_1, \dot{y}_1)^T$, a reduced system is formulated below.

$$\dot{\eta} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \eta + \begin{bmatrix} 0 \\ I \end{bmatrix} \mu = F\eta + G\mu \quad (3)$$

Solving the Riccati equation $F^T P + PF - PGG^T + Q = 0$ for $P = P^T$ and optimal solution for (3) can be obtained. To exponentially stabilize the system the control input μ must make the lyapunov function $V(\eta) = \eta^T P \eta$ to meet:

$$L_F V(\eta) + L_G(\eta)\mu \leq -\frac{\gamma}{\varepsilon} V(\eta) \quad (4)$$

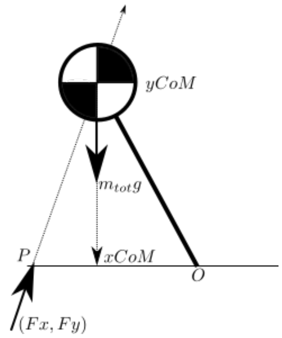

Fig. 1. LIPM model. The COP depends on the reaction forces on the floor and the COM.

In order to prevent the COP to lie beyond the foot support, a simplified model based on the linear inverted pendulum (LIPM) (Kajita et al. 2003) is considered to provide an approximation of the COP placement. The model is shown in Fig. 1, to compute the COP location (P) in the ground, moments at the CoM are taken which yields the following COP equation, $P = yCOM\frac{F_x}{F_y} + xCOM$, $xCOM$ and $yCOM$ are the COM coordinates along the horizontal and vertical line respectively, with respect to the inertial frame placed in O. The reaction force on the ground is equal to the weight of the robot in the LIPM model $F_y = m_{tot}g$, and the horizontal force is responsible for the horizontal acceleration of the COM, $F_x = m_{tot}(x\ddot{COM})$, leading to $P = (yCOM)(x\ddot{COM})/g + xCOM$.

Since the idea is to prevent the COP to lie outside the foot support, the objective is to drive the point P close to the foot origin O by making $P = 0$. Then, the $x\ddot{COM}$ evolves according to $x\ddot{COM} = -gxCOM/yCOM$.

Considering the full dynamics of the system, an output selected as $y_2 = xCOM$ can generate $x\ddot{COM}$ as:

$$\ddot{y_2} = x\ddot{COM} = L_{f^2}y_2(q_e) + L_g L_f y_2(q_e)\bar{u} \tag{5}$$

The controller can be expressed using a QP based optimization as:

$$\begin{aligned} min \qquad & \bar{u}^T H\bar{u} + N\bar{u} + p\delta_1^2 + q\delta_2^2 \\ (L_F V(\eta) + \frac{\gamma}{\varepsilon}V) + L_G(L_{f^2}y_1 + L_g L_f \bar{u}) &\leq \delta_1 \\ (L_{f^2}y_2 + \frac{g}{yCOM}xCOM) + L_g L_f y_2 \bar{u} &= \delta_2 \end{aligned} \tag{6}$$

where, $H = (L_g L_f y_1)^T (L_g L_f y_1)$, $N = 2(L_{f^2}y_1)^T (L_g L_f y_1)$

The last constraint of the optimization (6) is aimed to keep the COP inside foot support with no need to construct any COM trajectory or define a balance strategy beforehand.

Results

A simulation has been performed by using the monoped model and applying forces to the hip. A comparison of two controllers was carried out: i) one controller considered the stiff tracking of the joint objectives (Fig. 2 up) and ii) the second controller was the proposed controller controlled the COP location with additional joint tracking (Fig. 2 down).

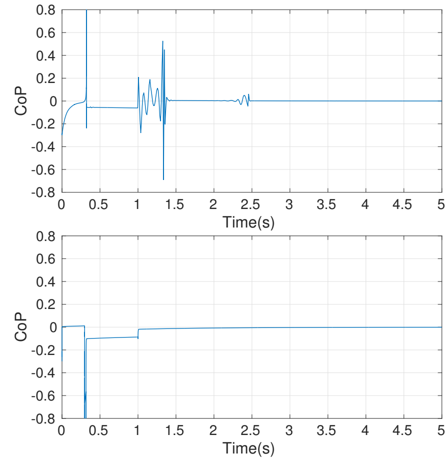

Fig. 2. Controllers outcome for directly joint tracking and joint tracking with COP manipulation respectively after a force (9N) is applied at the hip for 1s.

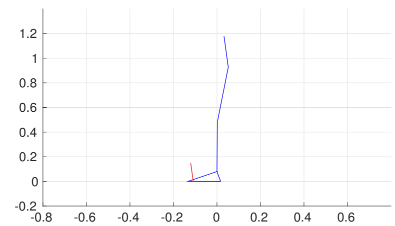

Fig. 3. Robot pose after disturbace of 9N at the hip in the positive horizontal direction. The red line indicates the reaction force acting on the COP.

Discussion

The proposed controller handles disturbances better than a controller that just tracks stiffly the desired joint locations. Our simplified model could give an appropiate estimation of the COP and effectively allow to control its location.

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