HUman Rehabilitation Group
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Push Recovery by Center of Pressure Manipulation

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Summary

+ Keep balance against perturbations
+ Combine Lyapunov techniques to the Inverted Pendulum paradigm
+ Joint Tracking and CoP manipulation in unified framework

Linear Inverted Pendulum

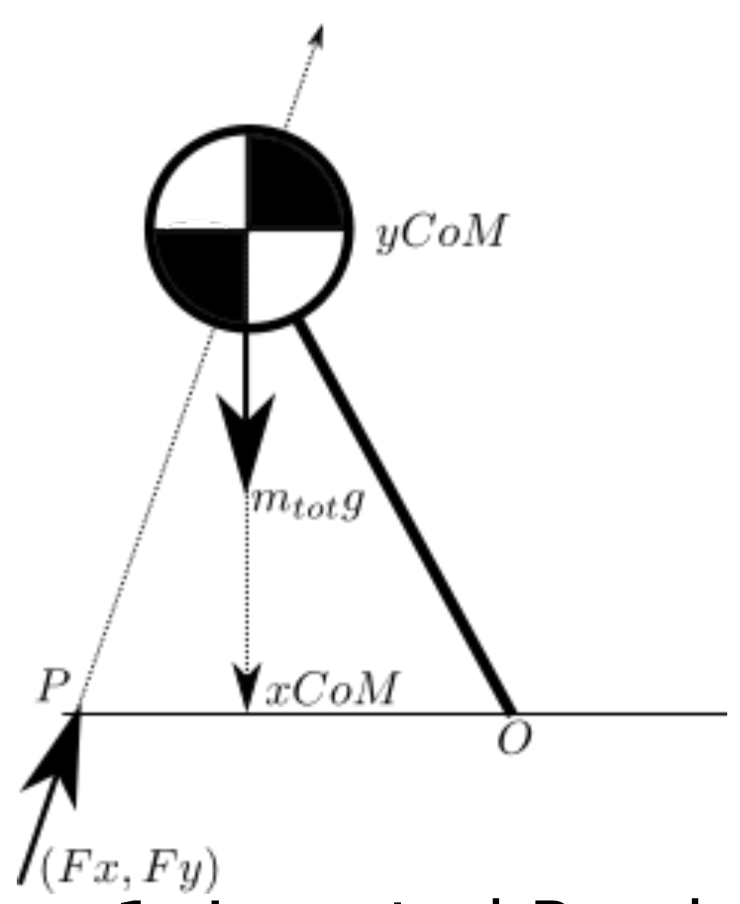

Figure 1. Inverted Pendulum.

$$P = (yCoM)(x\ddot{C}oM)/g + xCoM$$

Assuming that: $P = 0$

$$x\ddot{C}oM = -g(xCoM)/(yCoM)$$

This assumption is useful to consider the relative degree two output: $xCoM$

Dynamics

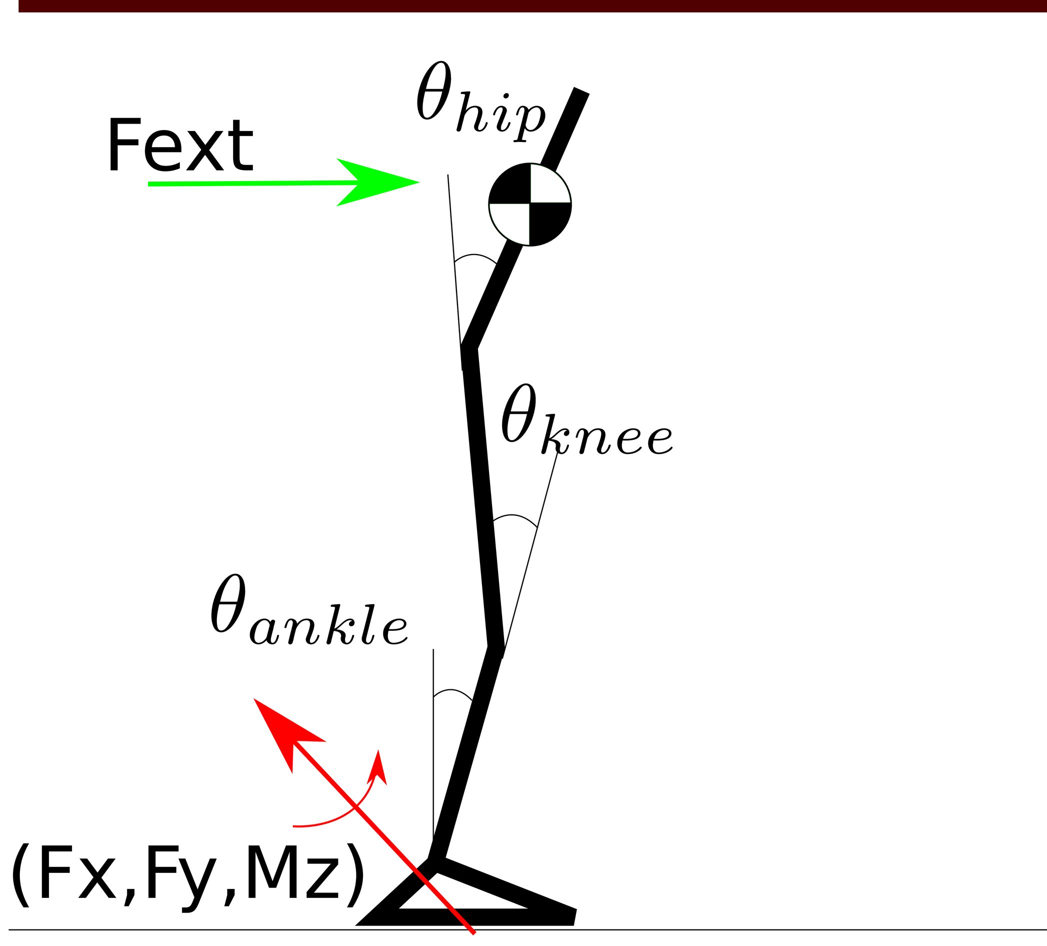

Figure 2. Robot model.

$$D_e(q_e)\ddot{q}_e + C_e(q_e,\dot{q}_e)\dot{q}_e + G_e(q_e)$$
$$= Bu + J_{sf}^T F = \bar{B}\bar{u}$$
$$q_e = (x, y, \theta_{ankle}, \theta_{knee}, \theta_{hip})$$

Nonlinear control

Pick:

Tracking
$$y_1(q_e) = (\theta_{ankle}, \theta_{knee}, \theta_{hip})$$

Center of Mass
$$y_2(q_e) = xCoM$$

From Feedback Linearization:
$$\ddot{y}_1 = L_f^2 y_1 + L_g L_f y_1 \bar{u} = \mu$$

Transverse Dynamics:
$$\eta = (y_1, \dot{y}_1)^T$$
$$\dot{\eta} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \eta + \begin{bmatrix} 0 \\ I \end{bmatrix} \mu = F\eta + G\mu$$

Using Riccati equation:
$$F^T P + PF - PGG^T + I = 0$$

Define a Lyapunov Function:
$$V(\eta) = \eta^T P \eta$$

Implementing ES-CLF control:
$$L_F V(\eta) + L_G(\eta)\mu \leq -\frac{\gamma}{\epsilon} V(\eta)$$

By making the xCoM acceleration to follow the evolution when P = 0, it is possible to drive to zero the center of pressure

$$\ddot{y}_2 = x\ddot{C}OM = L_f^2 y_2 + L_g L_f y_2 \bar{u}$$
$$x\ddot{C}oM = -g(xCoM)/(yCoM)$$
$$L_f^2 y_2 + L_g L_f y_2 \bar{u} = -g(xCoM)\ (yCoM)$$

Optimization based controller:
$$min \quad \bar{u}^T H \bar{u} + N\bar{u} + p\lambda_1^2 + q\lambda_2^2$$
$$(L_F V + \frac{\gamma}{\epsilon} V) + L_G V(L_f^2 y_1 + L_g L_f \bar{u}) \leq \lambda_1$$
$$L_f^2 y_2 + \frac{g}{yCOM} xCOM + L_g L_f y_2 \bar{u} = \lambda_2$$

where,
$$N = 2(L_f^2 y_1)^T (L_g L_f y_1)$$
$$H = (L_g L_f y_1)^T (L_g L_f y_1)$$

Tracking

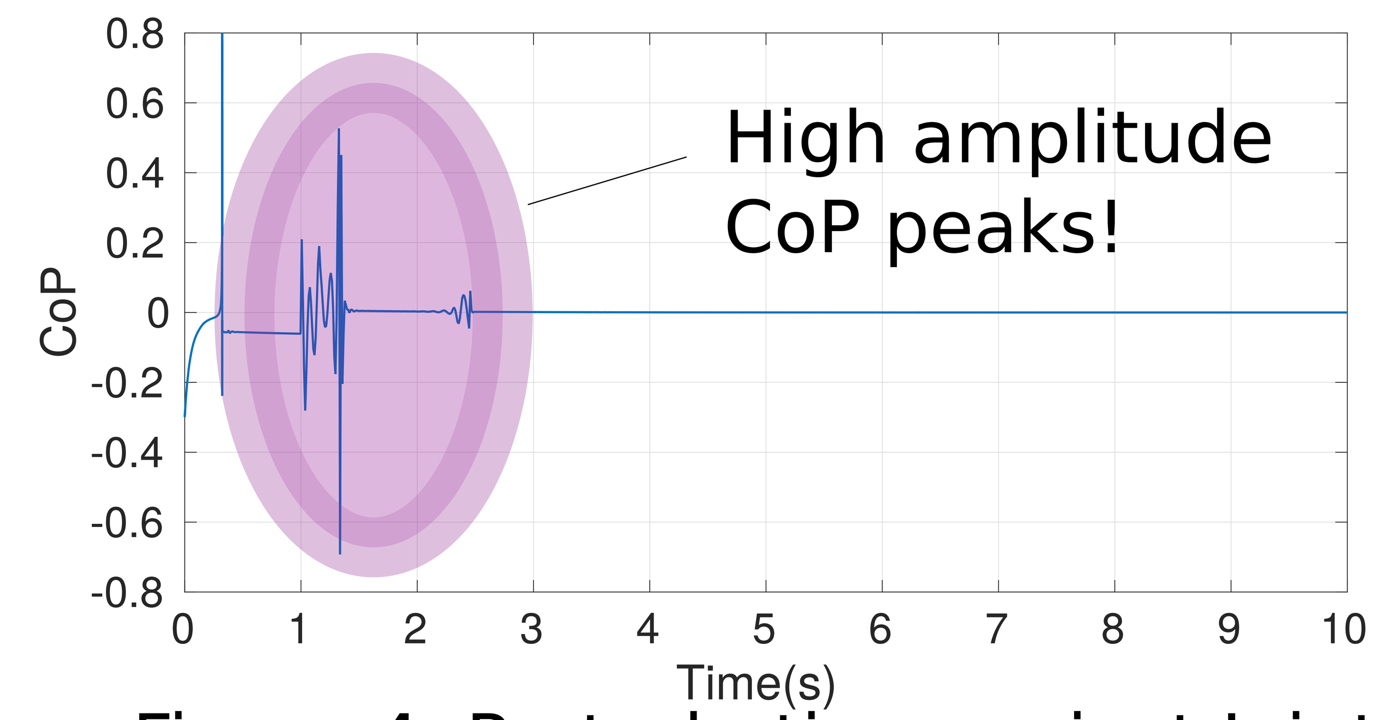

Figure 4. Perturbation against Joint Tracking robot.

Tracking + CoM

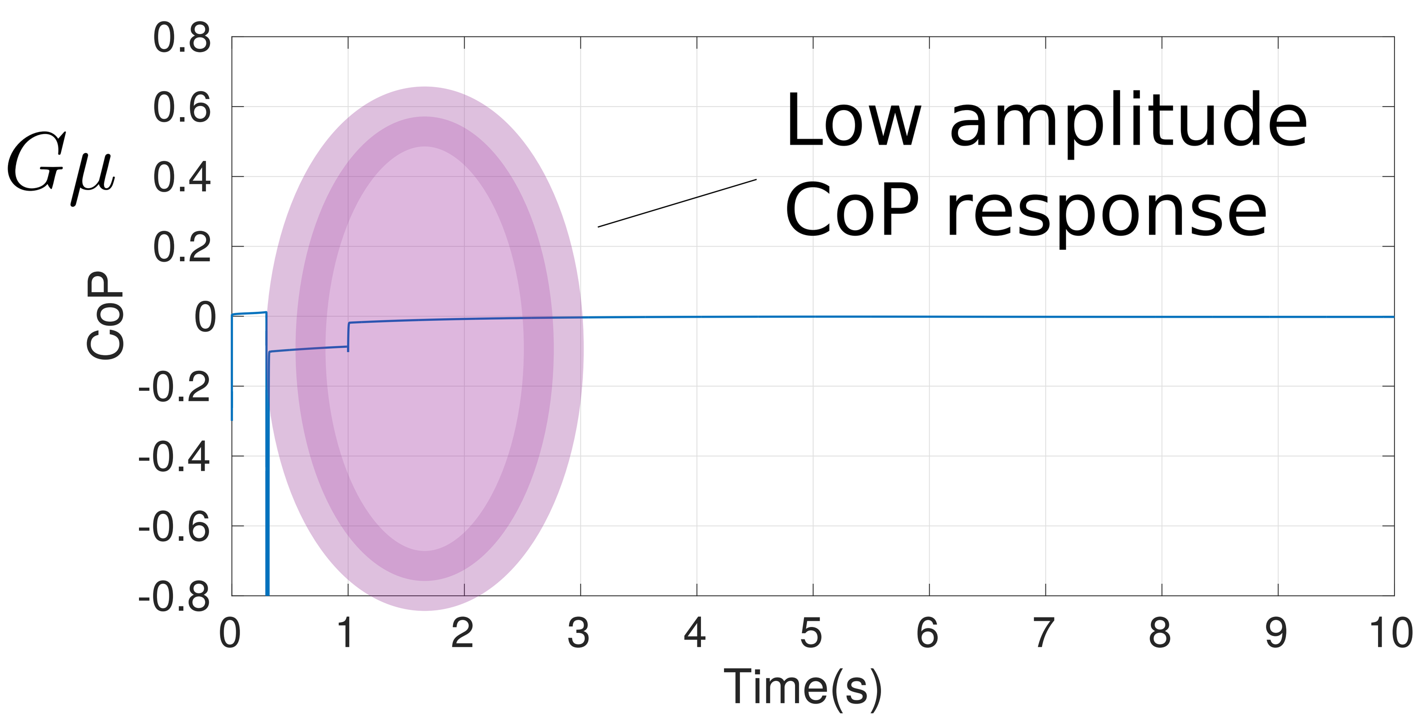

Figure 5. Perturbation against Joint Tracking + CoM Manipulation robot.

Conclusions

+ By modifying the CoM location according to (1) it is possible to indirectly control the CoP.
+ A QP based controller can be used to select appropriate control signals to keep balance and manipulate CoP position.

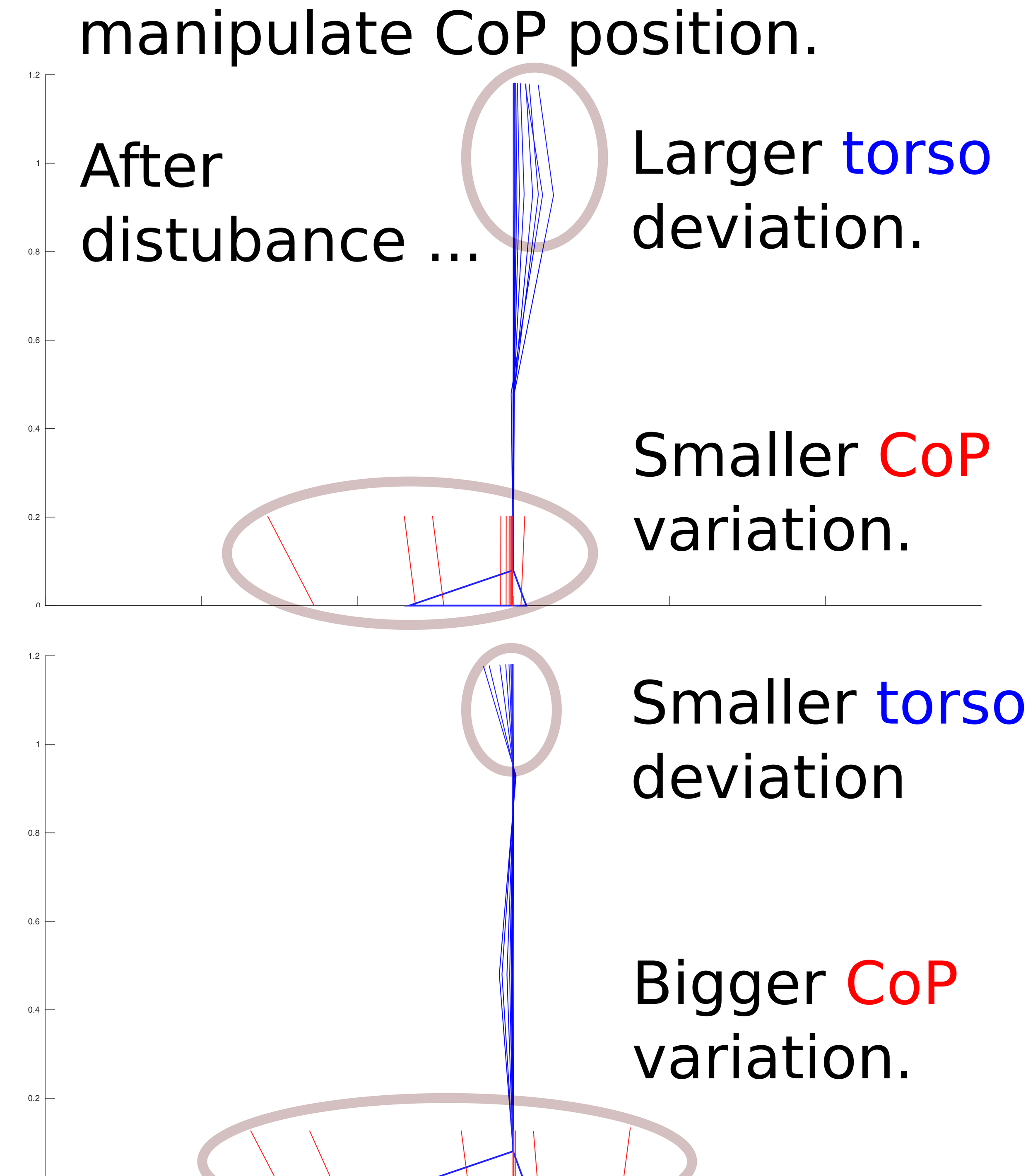

Figure 6. Comparision of controllers.

References

[1] Hofmann, Andreas. 2006. Robust execution of bipedal walking tasks from biomechanical principles.
[2] Kajita, Shuuji et al. 2003. Bipedal walking pattern generation by a simple three-dimensional inverted pendulum model. Advanced Robotics.
[3] Morris, Benjamin et al. 2013. Sufficient conditions for the lipschitz continuity of qp-based multi-objecti control of humanoid robots. Decision and control (CDC)
[4] Sinnet, Ryan. 2011. A human-inspired hybrid control approach to bipedal robotic walking. IFAC.
[5] Stephens, Benjamin. 2007. Humanoid Push recovery. Humanoid Robots, Intl conference on.