Modeling and analysis of posturographic data using Markov chains

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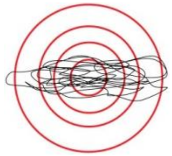
Overview

- Background
  - Center of Pressure
- Methods:
  - Traditional Measures: Descriptive and Stochastic measures
  - New Measures: Invariant Density Analysis
- Results
- Conclusions and Future Work

Postural sway : COP for quiet stance

- COM : center of mass (blue)
- COP : center of pressure (red)
  - Summation of the reaction forces underneath the subject's feet [1]
  - The most frequently studied measures to investigate stability and pattern of postural control

- There are several parameters for COP
  - Traditional stabilometric parameters [1]
  - Stochastic approaches [2,3]

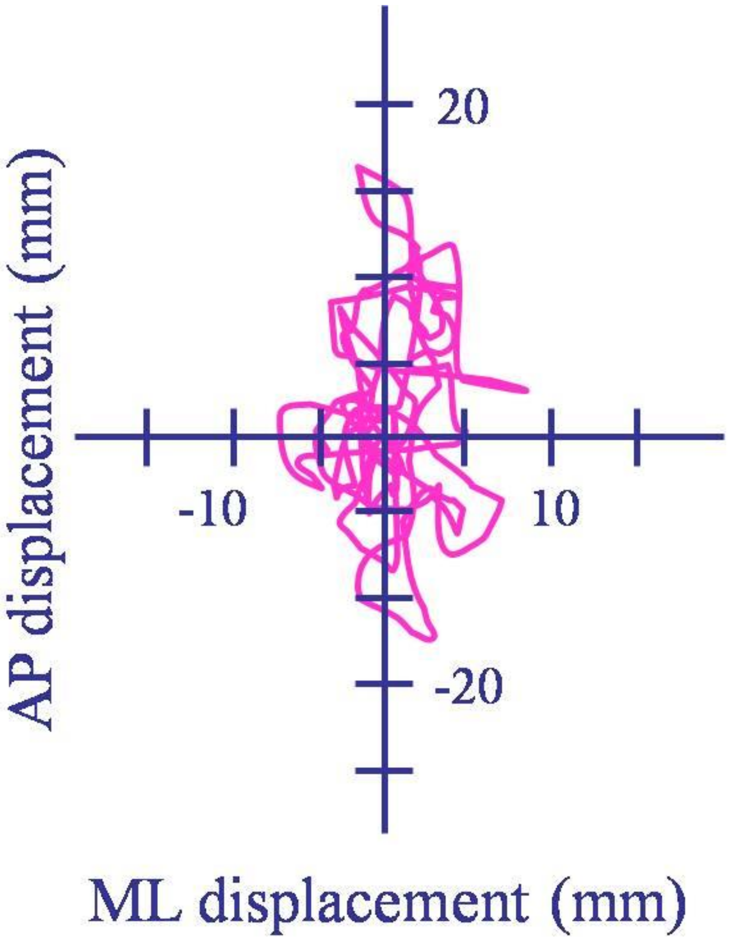


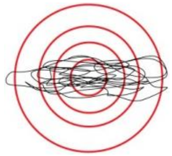
[1] Prieto, T.E. et. al., IEEE Trans Biomed Eng, Vol 43, 1996
[2] Collins, J.J., De Luca, C.J., Exp Brain Res, Vol 95, 1993
[3] Bosek, Maciej, and et al., Human Movement Science, 22, 2004

Traditional parameters

- Traditionally analyzed using measures to describe statistics of trajectory [1]
    - SD
    - Max distance
    - Range
    - Velocity
    - Path length
    - Sweep area

- Have questionable reliability [4]



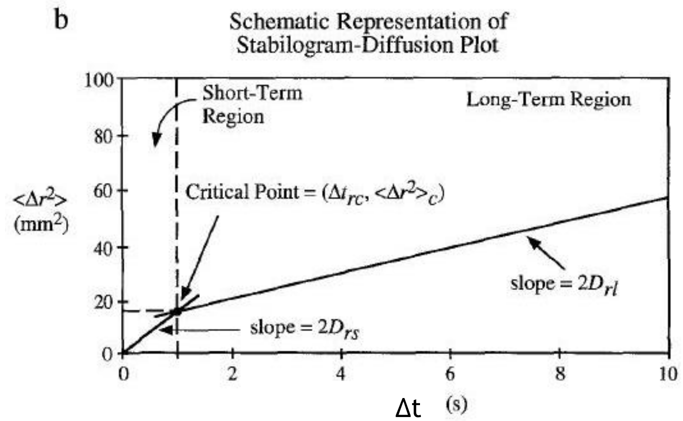


[1] Prieto, T.E. et. al., IEEE Trans Biomed Eng, Vol 43, 1996
[4] Doyle, T.L. et. al., Arch Phys Med Rehabil, Vol 86, October 2005

Stochastic approaches

Stabilogram diffusion analysis [2]

- COP Data as 1D or 2D random walks
- Applied diffusion analysis
- Open loop and closed loop control schemes



Statistical mechanics approach [3]

- Used Langevin equation for stochastic processes
- Calculated the coefficients of Langevin equation that best fit the COP data

Both have parameters of diffusion coefficients and scaling exponents

→ Hard to interpret for physical meaning

→ Can we develop a better stochastic-based method that can provide better physical insights?

[2] Collins, J.J., De Luca, C.J., Exp Brain Res, Vol 95, 1993.
[3] Bosek, Maciej, and et al., Human Movement Science, 22, 2004

A new metric : Markov chains

- In this study, we assume that COP data behave like Markov chains

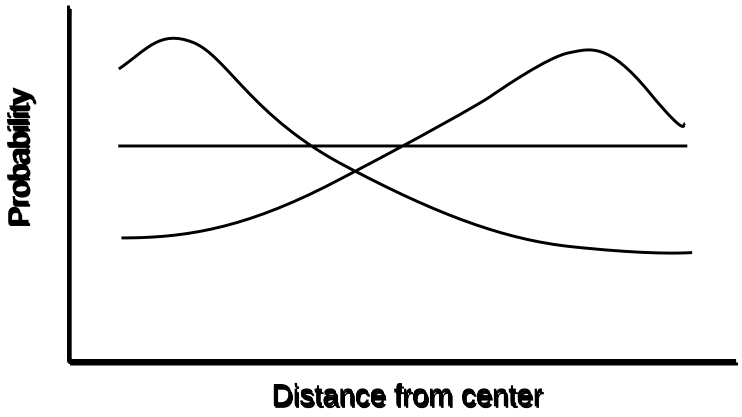
- Markov chains
  - Are stochastic processes $\rightarrow$ good to use when system is not deterministic
  - Have zero memory $\rightarrow$ almost all systems depend on the present states, not the past
  - Represent reduced order of dynamics of the system $\rightarrow$ approximates dynamics well
  - Have **invariant density** for most cases $\rightarrow$ predicts long-term behavior
  - Are intuitive

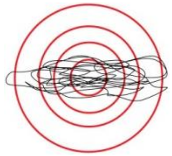
Probability distribution

It describes behavior of the dynamics of a stochastic system similar to how equations of motion describe a deterministic system. For example,

state ≡ distance of COP from center

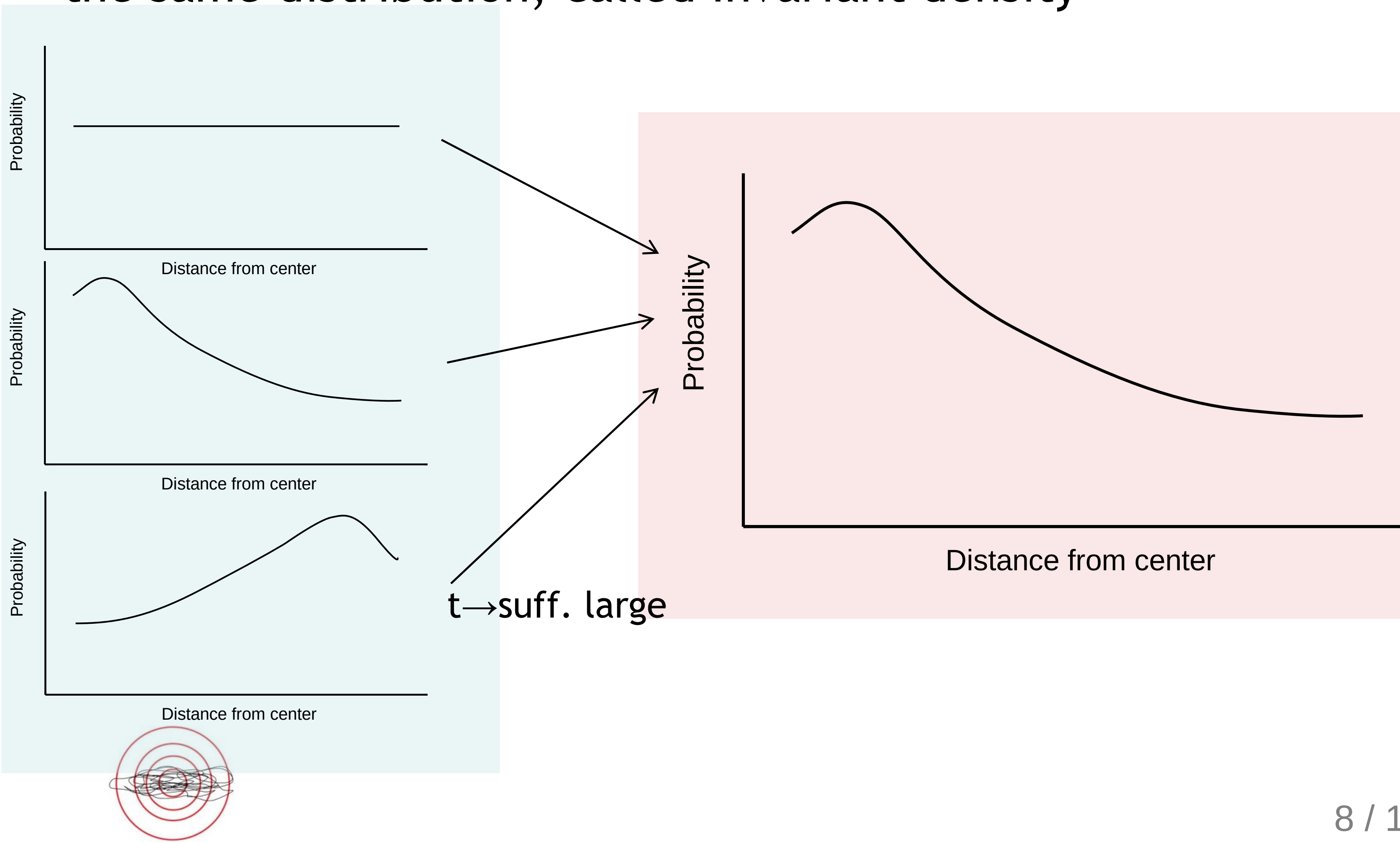
Distribution is given as





Invariant density (distribution)

Regardless of initial condition, we will eventually have the same distribution, called Invariant density

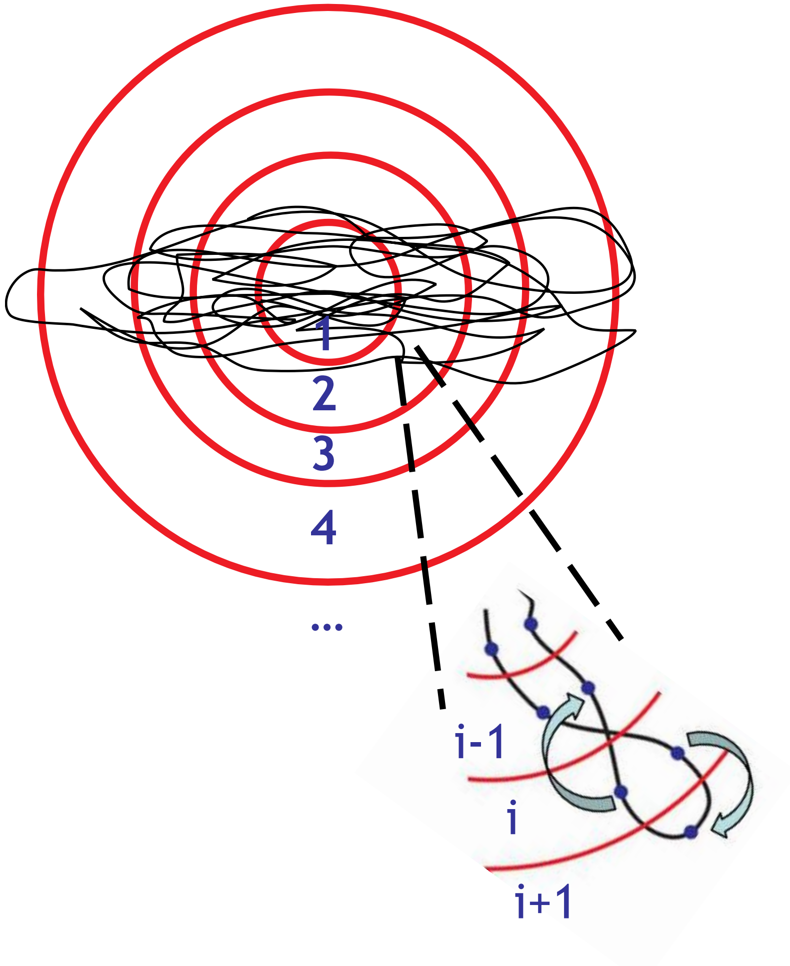


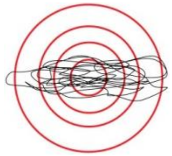
Invariant density analysis (IDA)

Calculating invariant density π

1) Find centroid of COP
   $\rightarrow$ zero mean adjustment
2) Define states as concentric rings emanating from the centroid
3) Calculate the <u>transition matrix</u>[1] P
4) Solve for the invariant density

$$\pi = \pi P$$





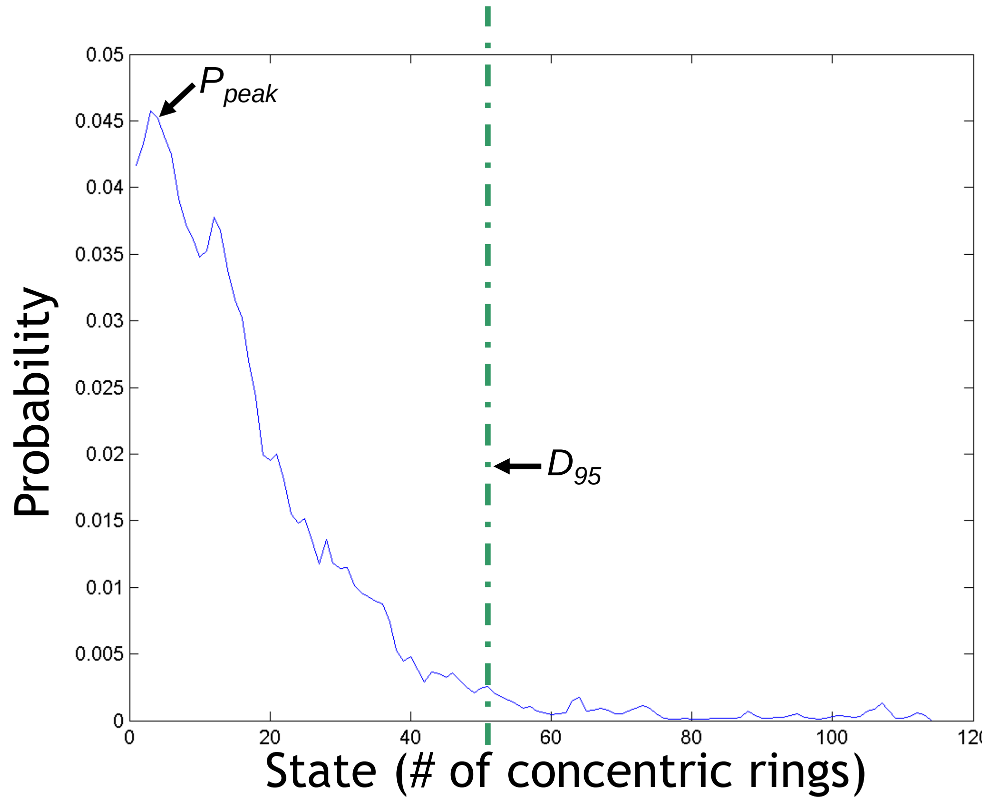
1) Transition matrix describes the probability of transition between states

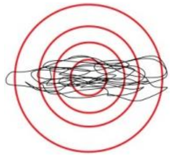
Invariant density analysis (IDA)

Four key parameters describing invariant density

- Peak probability (P_{peak})
  - How much COP concentrated
- Distance to 95% cumulative probability (D_{95})
  - How wide COP dispersed
- Second Eigenvalue (EV_2)
  - How fast COP reaches invariant distribution
- Entropy (H)
  - How different control schemes or learning effects are involved

$$H = \sum -\pi \log \pi$$





Low H	High H
Behavior is predictable Capable of control balance	Behavior is more random Less capable

Reliability and validity tests

Reliability test – Can we combine multiple short trials to get invariant density as one long trial? (One 5 min vs. ten 30 sec)

- Eight young adults
- Ten 30 sec quiet standing trials with eyes open
- One 5 min quiet standing trial with eyes open

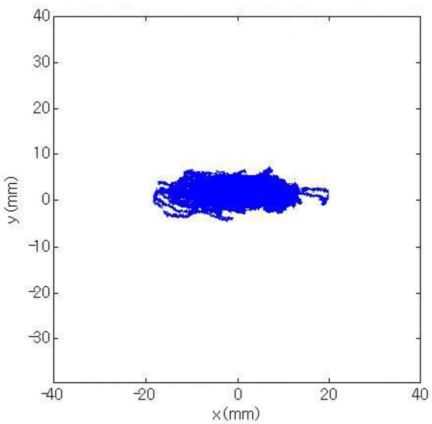
Validity test – Is this metric sensitive enough to distinguish groups?

- Three different age groups
  - young (N=15), middle-aged (N=15), old adults (N=15)
- Ten 30 sec quiet standing trials with eyes open

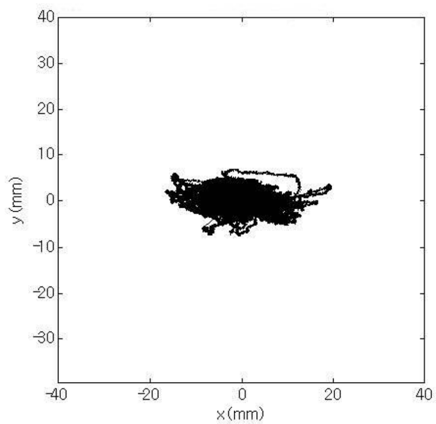
AMTI Forceplate with 1000 Hz

Reliability results

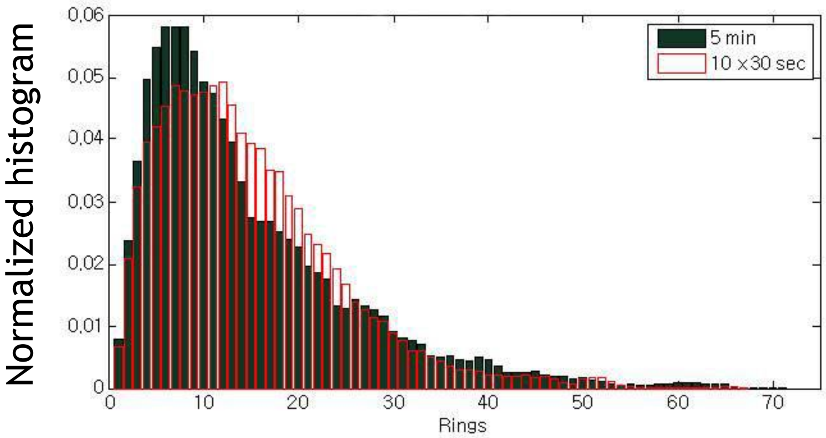
- One 5 min vs ten 30 sec
  - No significant differences by visual inspection



One 5 min trial COP

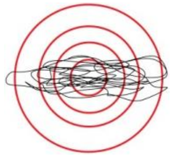


Ten 30 sec trials COP

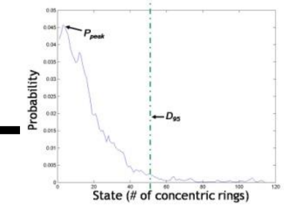


  - No statistically significant differences among four parameters

- We could use both one 5 min trial and ten 30 sec trials



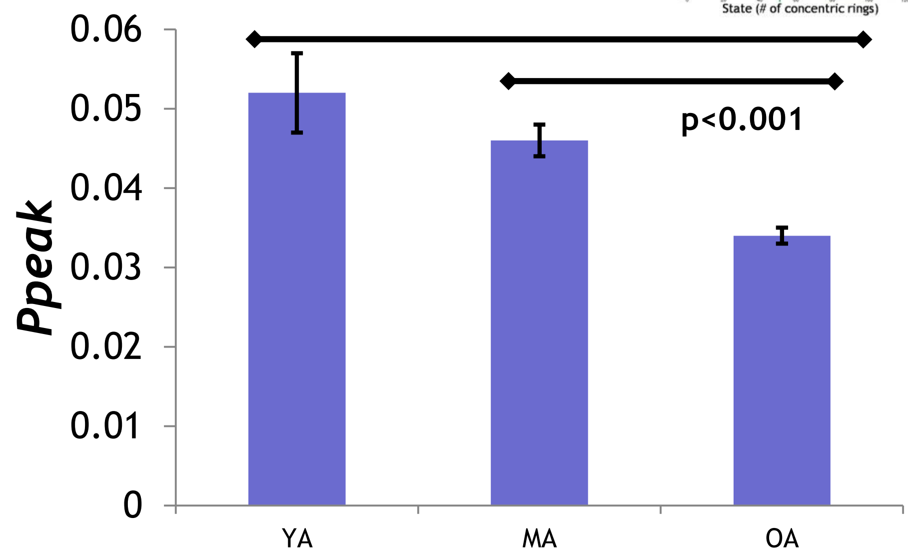
Validity results

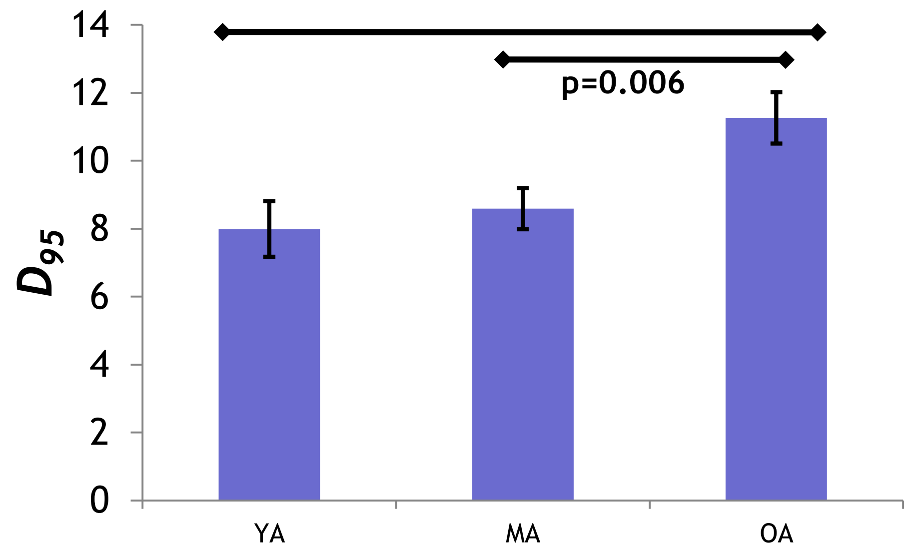


- All four parameters were statistically significantly different between age group

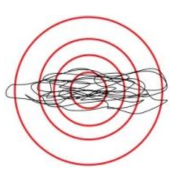
- YA have much more chances to stay near the center (P_{peak})

- OA are wandering around much wider (D_{95})

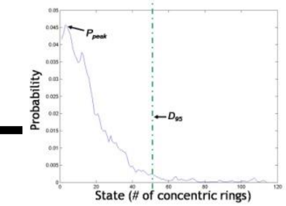




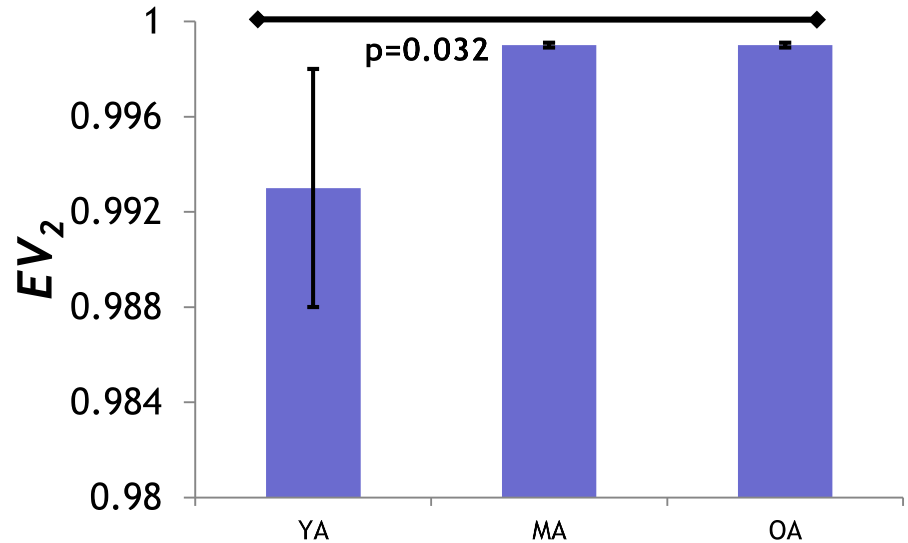
Error bar : standard error



Validity results

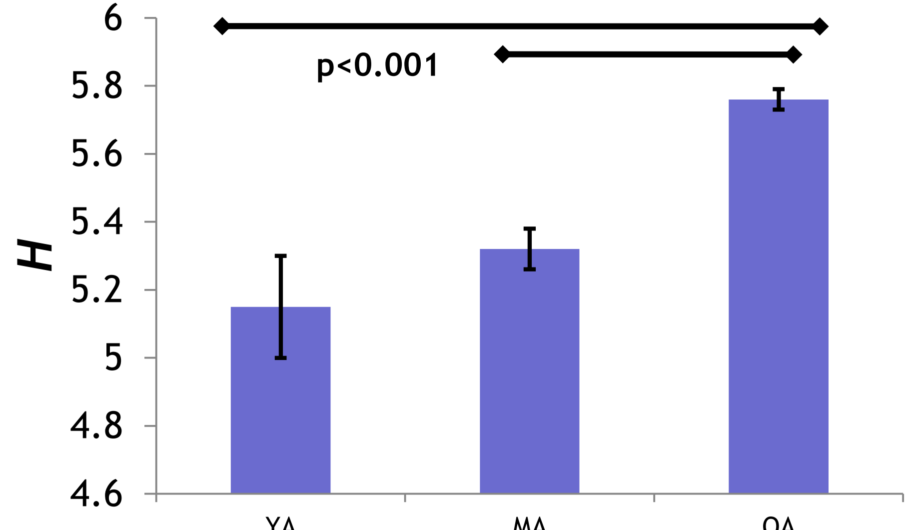


- YA are reaching invariant density much faster. Process for YA is more stable (EV_2) [5]



- Less information is needed to describe COP of YA. COP movement of YA is more predictable (H)

→ May suggest that YA are more capable of controlling and maintaining their postures



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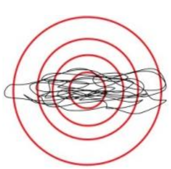


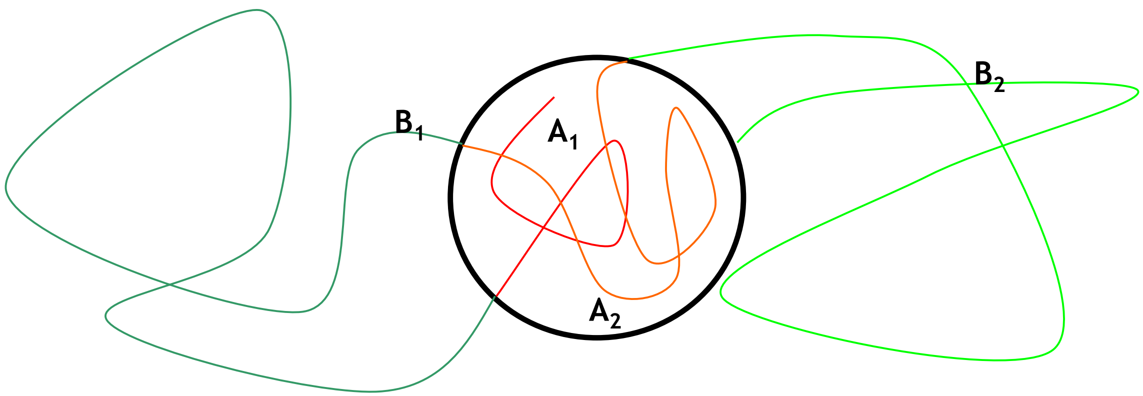
[5] Meyer, C.D., SIAM J Matrix Analysis and App Vol 15(3), 1994.

Conclusions

- We proposed a new and novel analysis tool for describing the COP dynamics
  - Invariant density of quiet standing COP measures can be used to differentiate between subject populations
  - Statistically significant differences were found between age groups in all four parameters (P_{peak}, D_{95}, EV_2, H)

Future Work

- Find transition point between open-loop and closed-loop control mechanism introduced in Collins work [2].
  → So far, we found it to be where the second Eigenvector crosses zero



- Analysis of stability, robustness (LF, PE) and performance performance bounds of COP
- Reconstruct COP trajectory using Markov chains

[2] Collins, J.J., De Luca, C.J., Exp Brain Res, Vol 95, 1993.

Acknowledgement

Thanks to Prof. Prashant Mehta for guidance and comments

Thank you

Questions?

What is Markov?

- A random variable X is Markov if
  - Future depends only on present

$$P(X_{n+1} = i_{n+1} \mid X_0 = i_0, X_1 = i_1, \cdots, X_n = i_n)$$
$$= P(X_{n+1} = i_{n+1} \mid X_n = i_n) = p_{i_n i_{n+1}}$$

  - Matrix $P = \{p_{ij}\}$ is stochastic

Transition matrix

- Every Markov chain has initial distribution (λ) and transition matrix (P)

$$P = \begin{array}{cc} \begin{bmatrix} \ddots & & & & & \\ & 0 & 1/3 & 2/3 & 0 & \\ & & & & & \ddots \end{bmatrix} & i^{th} \\ \qquad\qquad j^{th} & \end{array}$$

Evolution of dist. and invariant…

- Distribution evolves

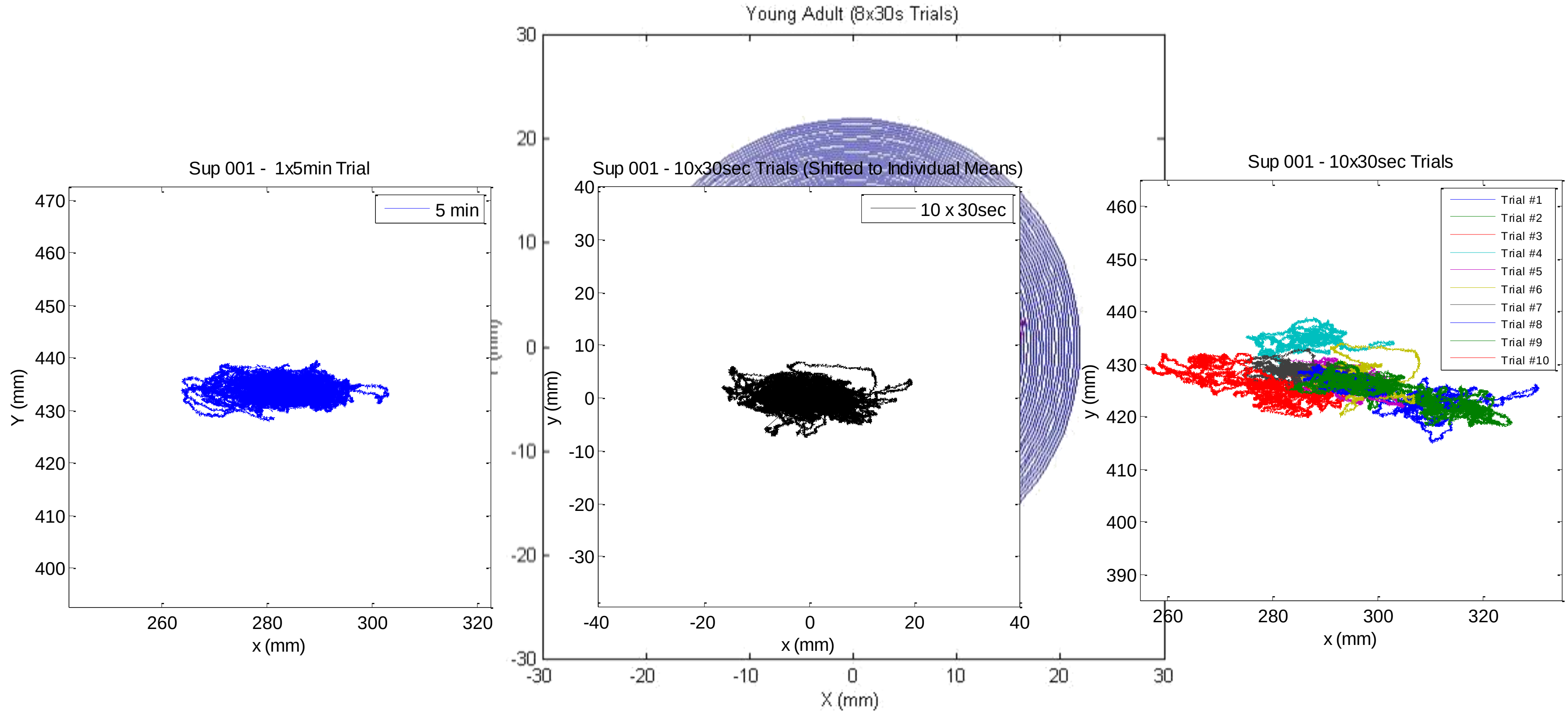
$$\lambda_{n+1} = \lambda_n P$$

- Distribution (π) is invariant if

$$\pi = \pi P$$

- Invariant distribution (or density) is right Eigenvector with Eigenvalue of 1

- Invariant distribution is long-term behavior. Short-term behavior is transient


Young Adult (8x30s Trials)
X (mm)
Sup 001 - 1x5min Trial
5 min
Y (mm)
x (mm)
Sup 001 - 10x30sec Trials (Shifted to Individual Means)
10 x 30sec
y (mm)
x (mm)
Sup 001 - 10x30sec Trials
Trial #1
Trial #2
Trial #3
Trial #4
Trial #5
Trial #6
Trial #7
Trial #8
Trial #9
Trial #10
y (mm)
x (mm)