ESTIMATING THE MOMENT OF INERTIA OF THE HUMAN BODY AS A SINGLE LINK INVERTED PENDULUM MODEL

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INTRODUCTION

Moment of inertia of the human body is a frequently used parameter to understand the human postural control system with control models that represent the upright body as a single link inverted pendulum. However, there is no unique way of calculating the moment of inertia. The most frequently used method is regarding the human body as a point mass. Then the moment of inertia can be easily computed as of $J=mh^2$, where m is mass of the body and h is height of the point mass from ankle. To obtain more accurate moment of inertia values, combining all moments of inertia of each segment based on anthropometric data can be used (Winter, 2005).

In this paper, a simple single link inverted pendulum model was used to derive equations of motion (EOM) relating center of mass (COM) and center of pressure (COP). COP was directly computed from forceplate data. COM was computed from the gravity line projection (GLP) method (Zatsiorsky and Duarte, 2000) which is robust and independent of standing conditions; furthermore, it only requires force plate data. By identifying the value of the moment of inertia, which best fits the COM from the EOM to the COM from GLP, the moment of inertia can be estimated.

METHODS

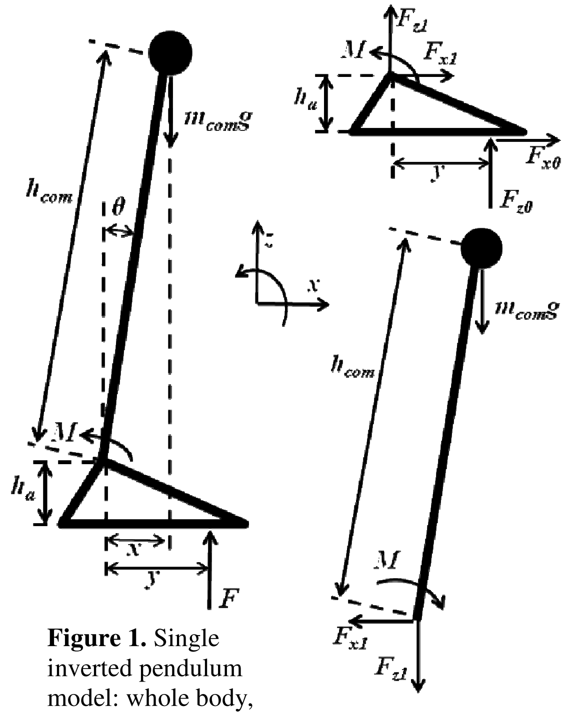

Figure 1. Single inverted pendulum model: whole body, and free body diagrams of foot and pendulum.

EOM of foot
By assuming that the foot is stationary,

$$\sum F_x = F_{x0} + F_{x1} = m_{foot}\ddot{x}_{foot} = 0 \tag{1}$$

$$\sum F_z = F_{z0} + F_{z1} = m_{foot}\ddot{z}_{foot} = 0 \tag{2}$$

$$\begin{aligned}\sum M_{ankle} &= M + yF_{z0} + h_a F_{x0} \\ &= J_{foot}\ddot{\theta}_{foot} = 0\end{aligned} \tag{3}$$

EOM of pendulum
By small angle approximation,

$$\sum F_x = -F_{x1} = m_{com}\ddot{x}_{com} \tag{4}$$

$$\sum F_z = -F_{z1} - m_{com}g = m_{com}\ddot{z}_{com} \tag{5}$$

$$\begin{aligned}\sum M_{ankle} &= -M - m_{com}gh_{com}\sin\theta \\ &= -M - m_{com}gh_{com}\theta = -J\ddot{\theta}\end{aligned} \tag{6}$$

Since $x_{com} = -h_{com}\theta$ by small angle approximation, together with (1), (3), (4) and (6), the following equation can be derived.

$$x_{com} = \frac{1}{m_{com}g}\left(\frac{J}{m_{com}h_{com}} - h_a\right)F_{x0} - y\frac{F_{z0}}{m_{com}g} \tag{7}$$

where, m_{com} is body mass without foot mass, g is gravity acceleration, h_{com} is length from ankle to COM and approximated from anthropometric data (0.559 of body height), h_a is ankle height from the ground, F_{x0} is anterior-posterior (AP) ground reaction force (GRF), F_{z0} is vertical GRF, and y is COP in AP direction. Both forces and y can be measured from the forceplate.

Parameter identification

COP in the AP direction can be computed from GLP method (Zatsiorsky and Duarte, 2000). The moment of inertia, J, in (7) can be therefore be identified by minimizing the following objective function.

$$\min F = \left\| x_{com,GLP} - x_{com,eq(7)} \right\|^2 \tag{8}$$

Experiment

Thirty healthy adult subjects were tested during a single session. Each conducted ten quiet standing trials, 30s in duration. The subject was instructed to maintain a quiet, upright posture throughout the recording. The subject stood on a force plate (AMTI, BP600900) with arms crossed at the chest and eyes open.

RESULTS

The results showed that the identified moments of inertia are not the same as the point mass configuration. It was found that there was relatively high correlation (r=0.85) between the point mass moment of inertia and the identified moment of inertia. Roughly, there was a constant ratio of 0.9 between the identified moment of inertia and point mass moment of inertia. However, this ratio changed as body mass index (BMI) changed with relatively weak correlation (r=0.43). It was hard to find a single equation that relates moment of inertia, body mass and body height.

SUMMARY/CONCLUSIONS

A simple way of estimating the moment of inertia was investigated. With a single link inverted pendulum model, the EOM was derived, and a relation between COM and COP was found. With parameter identification by minimizing the norm of errors of COM locations from the model and GLP, the moment of inertia can be estimated.

REFERENCES

Winter, D.A. (2005) *Biomechanics and motor control of human movement, Wiley 3rd ed.*

Zatsiorsky, V.M. and Duarte, M, *Motor Control* 4, 185-200, 2000

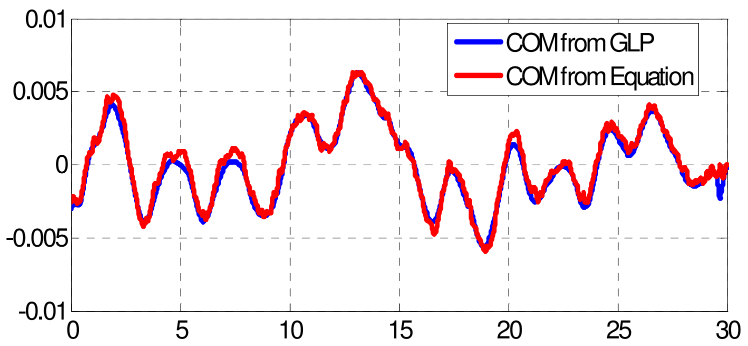

Figure 2. Sample COM plots computed from GLP and equation (7) during quiet standing. This figure shows COM plots when the moment of inertia was optimized by minimizing the objective function (8).