Estimating the moment of inertia of the human body as a single link inverted pendulum model

Pilwon Hur and Elizabeth T. Hsiao-Wecksler

Department of Mechanical Sciences & Engineering
Human Dynamics and Controls Laboratory
University of Illinois at Urbana-Champaign, Urbana, IL, USA
E-mail: phur2@illinois.edu   Web: www.mechse.illinois.edu/research/hsiao-weckesler/

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INTRODUCTION

- Moment of inertia of the human body is a frequently used parameter to understand the human postural control system with control models that represent the upright body as a single link inverted pendulum.
- However, there is no unique way of calculating the moment of inertia of the entire body.
- The most frequently used method is regarding the whole human body as a point mass, e.g., $J = mh^2$.
- Sometimes, anthropometric data are used to compute more accurate total moment of inertia from individual segments [1].
- In this study, the moment of inertia of the whole body was estimated by an optimization which minimized center of mass (COM) derived from equation of motion (EOM) of a single link inverted pendulum model to fit the experimentally determined COM.

METHODS

Mathematical model

- The human postural control system can be simplified by a single link inverted pendulum.

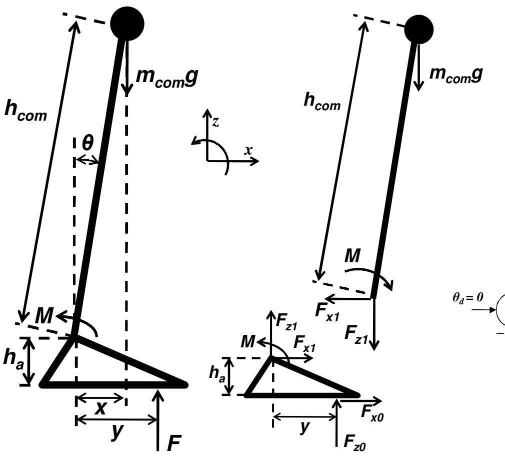

Fig. 1 Single link inverted pendulum model: free body diagrams of whole body, foot, and pendulum

- From the EOMs derived from the free body diagrams (Fig. 1) with assumption of stationary foot and small angle approximation, the equation for COM horizontal position can be derived as:

$$x_{com,EOM} = \frac{1}{m_{com}g}\left(\frac{J}{m_{com}h_{com}} - h_a\right)F_{x0} - y\frac{F_{z0}}{m_{com}g}$$

- Note that all parameters are measurable except the moment of inertia, *J*.

Determination of COM from force plate data

- Since it is hard to directly measure the true COM, it can be estimated by modified gravity-line projection method [2-3].
- This method can derive COM horizontal position from AP force and COP data. Lean angle of the COM (θ) was then computed.
- The point of this method is that when the AP GRF becomes zero, AP COP and COM coincide. Therefore, by double integration of AP acceleration, AP COM can be estimated.

Optimization

- The moment of inertia was estimated such that the L2 Norm of the difference in AP COM positions is minimized. That is:

$$J = \arg\min_J \left\| x_{com,GLP} - x_{com,EOM} \right\|^2 \quad (1)$$

Validation by simulation

- The method proposed in this study was verified by mathematical simulation.
- A postural control model with a single link inverted pendulum model that was modulated by a time-delayed PD controller and passive torque generator was used [4] (Fig. 2). From the model, we computed AP COM, COP, GRF and J, which we assume to be "true" values.
- Model information is given as:
  mass m: 70 kg
  height of COM h: 1 m
  point mass moment of inertia $J = mh^2$: 70kg m²
  4 different moments of inertia representing different body shapes but with same m and h
  (J = 72.73, 77.89, 79.57, 87.36)
- Optimization with eq. 1 was performed to identify moment of inertia.

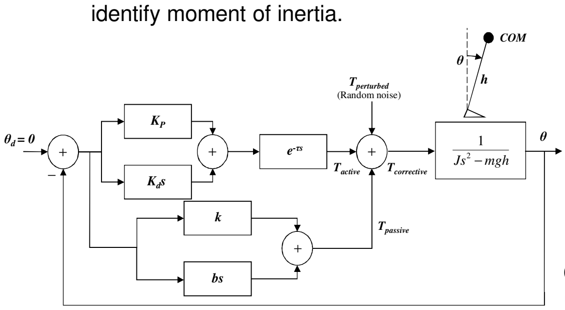

Fig. 2 Block diagram of postural control system

Experimental Protocol

- This technique was applied to experimental quiet stance data (ten 30s trials).
- 30 healthy adult subjects participated.
- The subject was instructed to maintain a quiet, upright posture with arms crossed at the chest and eyes open (Fig. 3).
- GRF and COP were recorded with a force plate (AMTI, BP600900), sampled at 1000 Hz.

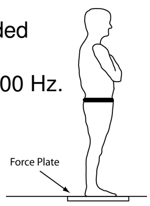

Fig. 3 Experimental setup

RESULTS

- *Mathematical simulation results* suggest that this optimization approach for determining whole body moment of inertia produces more accurate estimates than only assuming a point mass model of $J = mh^2$ (Table 1).

***Table 1** Optimized moments of inertia for four different J, assuming m = 70 kg, h = 1 m but different body shapes. Note point mass moment of inertia is J=mh² = 70 kg m².*

Known J	72.73	77.89	79.57	87.36
Optimized J for 10 trials	71.848 ± 0.195 (SD)	77.124 ± 0.084	78.720 ± 0.197	86.590 ± 0.214

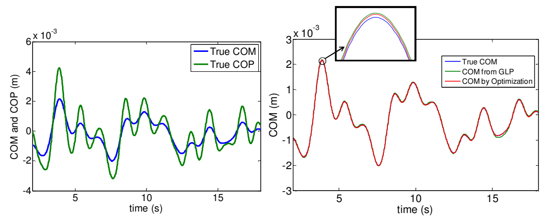

***Fig. 4** Sample true AP COM and COP computed from simulation (left), and AP COM computed from GLP and optimization methods using simulation data (right)*

- *Experimental results* indicate that moments of inertia computed by the point mass assumption model are not the same as the values computed by our optimization approach (Fig. 5).

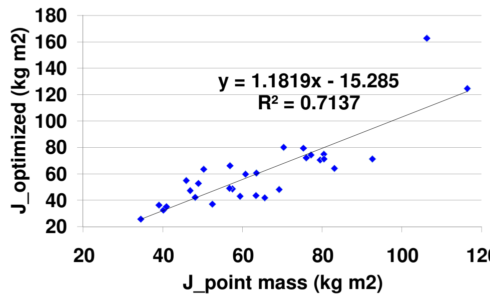

***Fig. 5** Experimental results of moments of inertia computed by point mass assumption and optimization approach.*

CONCLUSIONS

- We proposed an algorithm that can estimate the whole body moment of inertia during upright stance using a single link pendulum model.
- We found from the simulation study that the point mass approximation to estimate whole body moment of inertia is not appropriate.
- The proposed algorithm appears to estimate the whole body moment of inertia quite well.

References:

[1] Winter, D.A. *Biomechanics and motor control of human movement*, 2005.
[2] Zatsiorsky VM, King D, *J Biomech* 31: 161-164,1997.
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[4] Hur, P., et al. *ASB* P9-18, 2007.