A Step Towards Generating Human-Like Walking Gait via Trajectory Optimization through Contact for a Bipedal Robot with One-Sided Springs on Toes

Kenneth Chao and Pilwon Hur

09/27/17
Outline

• Motivation and background

• Trajectory optimization through contact

• Modified framework
  – Hermite-Simpson method
  – Cost function for virtual flexible components
  – Complementary constraints for one-sided springs

• Result, conclusion and future work
Motivation and background

Generating an efficient, robust walking gait with multiple contact domains is challenging

• The floating-based system is complex
  – High dimensions and nonlinearity
  – Non-convex constraints in kinematics and dynamics

• Existence of contact impact

• Combination of different contact domains
  – Different contact conditions (constraints) result in different dynamics
  – Mixture of full, over, or under actuation
Motivation and background

- Optimal control and trajectory optimization

**closed-loop solution** \( u = u(x) \)  
**open-loop solution** \( u = u(t) \)

- Global optimal solution
  - Hard to compute
  - Lower dimensional problems

- Local method, suboptimal solution
  - Easier to compute
  - Higher dimensional problems
Motivation and background

- Trajectory optimization is the collection of methods used to find the best choice of trajectory: $x(t)$ and $u(t)$
  - Single shooting
  - Multiple shooting
  - Direct collocation

  More accurate, harder to pose and solve

  Less accurate, easier to pose and solve

Continuous functions of time $\rightarrow$ Discrete set of real numbers

\[
\begin{align*}
t & \rightarrow [t_0, t_1, \ldots, t_N] \\
x(t) & \rightarrow [x_0, x_1, \ldots, x_N] \\
u(t) & \rightarrow [u_0, u_1, \ldots, u_N]
\end{align*}
\]

\[
\begin{align*}
t & = \text{grid point number} \\
x_k & = x(t_k) \\
u_k & = u(t_k)
\end{align*}
\]
State of the arts: Trajectory optimization for walking motion generation with multiple contact domains

- Hybrid Zero Dynamics (HZD)-based trajectory optimization with direct collocation (Ames et al. 2015-16)

- Trajectory optimization through contact (Posa et al. 2012)
State of the arts: Trajectory optimization for walking motion generation

<table>
<thead>
<tr>
<th>HZD-based</th>
<th>Optimization through contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trajectory optimization approach</td>
<td>Direct collocation method</td>
</tr>
</tbody>
</table>

Minimizing cost of transport (CoT)

\[ CoT(x) = \frac{\text{Energy}}{\text{Weight} \times \text{Distance}} = \frac{1}{mgd} \sum_{k=1}^{N} \sum_{i} | u_{k,i} \dot{q}_{k,i} | \]
State of the arts: Trajectory optimization for walking motion generation

<table>
<thead>
<tr>
<th></th>
<th>HZD-based</th>
<th>Optimization through contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact handling</td>
<td>Hybrid-invariant constraints for phase-switchings</td>
<td>Time-stepping method</td>
</tr>
<tr>
<td>Predefined contact sequence</td>
<td>Required</td>
<td>Not required</td>
</tr>
<tr>
<td>Result sensitivity to initial guess</td>
<td>Relatively low</td>
<td>Relatively high</td>
</tr>
<tr>
<td>Transcription method for collocation</td>
<td>Hermite-Simpson method (cubic splines)</td>
<td>Trapezoid method (linear functions)</td>
</tr>
</tbody>
</table>
Trajectory optimization through contact

\[ x = \{ h, x_0, \ldots, x_N, u_1, \ldots, u_N, \lambda_1, \ldots, \lambda_N \} \]

\[ \text{minimize} \quad CoT(x) \]

Complementary constraints for contact, e.g.

\[ G(x) \geq 0 \]
\[ H(x) \geq 0 \]
\[ G(x)^T H(x) = 0 \]

Kinematic, dynamic constraints, and periodic constraints

Modified framework

- Hermite-Simpson method
- Cost function with virtual flexible components
- Complementary constraints for one-sided springs
Modified framework (1/2)

- Hermite-Simpson method

- Cost function with virtual flexible components

\[
\text{cost}(x) = \frac{1}{mgd} \sum_{k=1}^{N} \sum_{i} |u_{k,i} q_{k,i}| + \omega \sum_{k=1}^{N} u_{k}^T u_{k}
\]

\[
\text{CoT}(x)
\]

\[
\omega \sum_{k=1}^{N} (u_{k} - kB_{k}q_{k})^T (u_{k} - kB_{k}q_{k})
\]
Modified framework (2/2)

- Complementary constraints for one-sided springs

\[ G(x) \geq 0 \]
\[ H(x) \geq 0 \]
\[ G(x)^T H(x) = 0 \]

\[ k_{toe}\theta_{toe} = (T_1 + T_2) - T^- \]
\[ (T_1 + T_2)T^- = 0 \]
\[ T_1T_2 = 0 \]
\[ \phi_{z,\text{toe}}T_1 = 0 \]
\[ \phi_{z,\text{toe}}, T_1, T_2, T^- \geq 0 \]

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Bu + J^T\lambda - J_{\theta,\text{toe}}^T T_1 \]
Software and Hardware

Hardware (robot model)
Planer bipedal robot Amber 3
  - Height: 1.45 m
  - Weight: 33.4 Kg
  - 7-link, 6 DOFs

Software
  - MATLAB and mathematica
  - Opt Solver
    - IPOpt
    - fmincon in MATLAB
Optimization overview

Opt. Problem Summary
- Time duration: a half gait cycle
- 31 (discretized) collocation points
- 1334 free variables, 1709 constraints
- Can be solved in 1-6 minutes*

Initial guess: ZMP-based flat-feet walking gait

Testing different set of contact constraints
- Sliding allowed contact constraints (SACC)
- Non-sliding contact constraints (NSCC)
- NSCC with one-sided springs (OSS)
Simulation Result (1/2)

<table>
<thead>
<tr>
<th></th>
<th>Initial guess</th>
<th>SACC</th>
<th>NSCC</th>
<th>OSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>0.577</td>
<td>0.048</td>
<td>0.049</td>
<td>2.644</td>
</tr>
<tr>
<td>Step length (m)</td>
<td>0.2</td>
<td>1.10</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Simulation Result (2/2)

\[
\min_X \sum (q_k - q_{k,\text{ref}})^T (q_k - q_{k,\text{ref}})
\]
\[
s.t. \quad \text{toe}_{\text{height}} \geq f(\text{toe}_{\text{horizontal}})
\]
\[
\text{heel}_{\text{height}} \geq f(\text{heel}_{\text{horizontal}})
\]
Hermite-Simpson constraints

Initial guess

Optimization through contact with SACC

Cost \approx 0.5

Cost \approx 0.047

Modified result from a kinematics-based optimization
Joint trajectories comparison to human data


Knee joint
Ankle joint
Hip joint
Current Experiment Result
(Not completely successful)
Conclusion and Future works

- The modified framework increases the accuracy of kinematic and dynamic approximation, and several schemes are provided and tested altering the walking behavior.

- Further modifications of model and constraints (e.g. for increasing foot clearance) are required for more natural generated walking gait.

- A better opt solver for handling optimization with complementary constraints and its relaxations is required.

- Implementations on bipedal robot and lower limb prosthesis as validation.
Thank you for your attention!

• Q&A